Math I

Unit-1.1 Set Theory

Set is a collection of well defined and distinct objects.

POINTS TO KNOW

- n(A∪B) is the number of elements present in either of the sets A or B.
- n(A∩B) is the number of elements present in both the sets A and B.

FORMULAS

For two sets A and B

- $n(A \cup B) = n(A) + (n(B) – n(A \cap B)$

For three sets A, B and C

- $n(A \cup B \cup C) = n(A) + n(B) + n(C) – n(A \cap B) – n(B \cap C) – n(C \cap A) + n(A \cap B \cap C)$

SOME VEEN DI AGRAM REPRESENTATIONS:

In above diagram set A is the subset of set B

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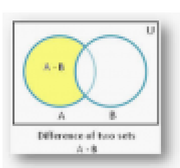
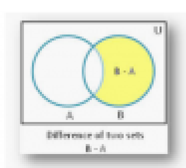
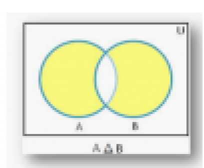




Unit 1.2 Real Number

N FACT

Nearly any number you can think of is a Real Number.

Real Numbers include:

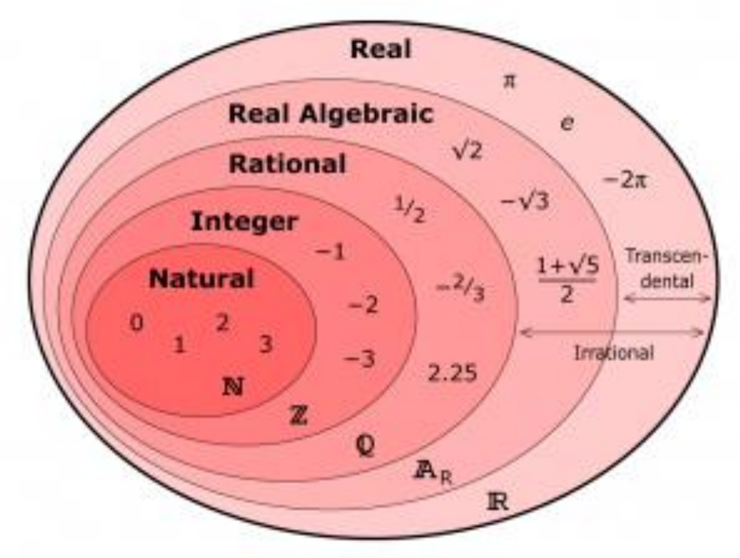
- Whole Numbers (like 0, 1, 2, 3, 4, etc)
- Rational Numbers (like 3/4, 0.125, 0.333..., 1.1, etc)
- Irrational Numbers (like **π**, √2, etc)

Real Numbers can also be positive, negative or zero.

So what is NOT a Real Number?

- Imaginary Numbers like **√−1** (the square root of minus 1) are not Real Number
- Infinity is not a Real Number

This diagram may help you to understand better:



real number hierarchy

Unit 1.3 Complex Number

A complex number is any number which can be written as a+*i*b where a and b are real numbers and *i*=√−1 is an imaginary number.
a is the real part of the complex number and *b* is the imaginary part of the complex number.
Example for a complex number: 9 + i2

Properties of Cube Roots of Unity

(1) Cube Roots of Unity are in G.P.

(2) Each complex cube root of unity is the square of the other complex cube root of unity.

Example: $w = \frac{-1+\sqrt{3}i}{2}$, $w^2 = \frac{-1-\sqrt{3}i}{2}$

(3) $1 + w + w^2 = 0$

(4) Product of all cube roots of unity = 1
i.e., $w^3 = 1$

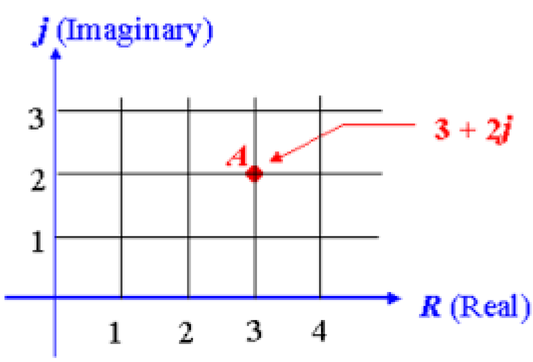
(5) $\frac{1}{w} = w^2$ and $\frac{1}{w^2} = w$

complex number

<u>POINTS TO KNOW</u>

- If the complex number a+ib=x+iy, then a=xand b=
- Fourth Roots of Unity, (1)1/4 are +1, -1, +i, -i

<u>This may help you to identify real and imaginary numbers</u>



complex_number_graph

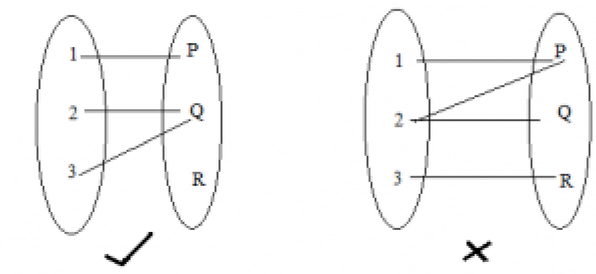
Unit 2 Relation, Function and Graph

A rule which associates each element of the set (A) with at least one element in set (B).

Function:

A function is a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output.

Eg:



function

A Condition for a Function:

Set **A** and Set **B** should be non-empty.

In a function, a particular input is given to get a particular output. So, A function **f: A->B** denotes that f is a function from **A** to **B**, where **A** is a domain and **B** is a co-domain.

- For an element, **a**, which belongs to **A**, a∊A, a unique element **b**, b∊B is there such that (a,b)∊ f.

The unique element **b** to which **f** relates **a**, is denoted by **f(a)** and is called **f of a**, or the value of **f at a**, or **the image of a under f.**

- The *range* of **f** (image of **a** under f)
- It is the set of all values of **f(x)** taken together.
- **Range of f** = { y ∊ Y | y = f (x), for some x in X}

A real-valued function has either **P** or any one of its subsets as its range. Further, if its domain is also either **P** or a subset of **P**, it is called a **real function**.

Representation of Functions

Functions are generally represented as f(x)

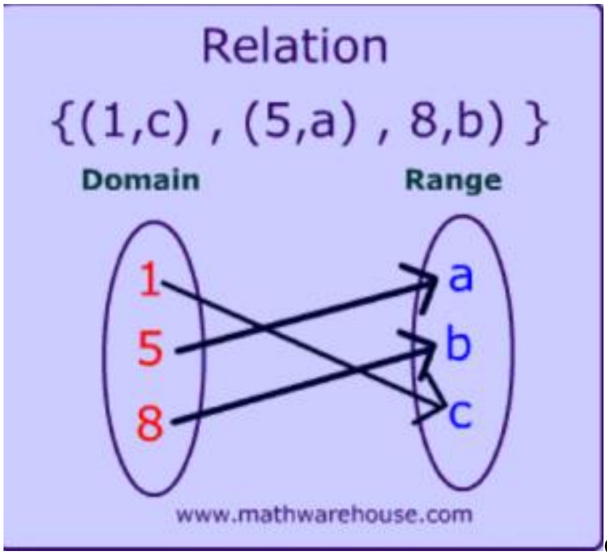
Let , f(x)=x3

It is said as f of x is equal to x cube.

Functions can also be represented by g(), t(),… etc.

Domain & Range

The **domain** of a function f(x) is the set of all values for which the function is defined, and the **range** of the function is the set of all values that ff takes.

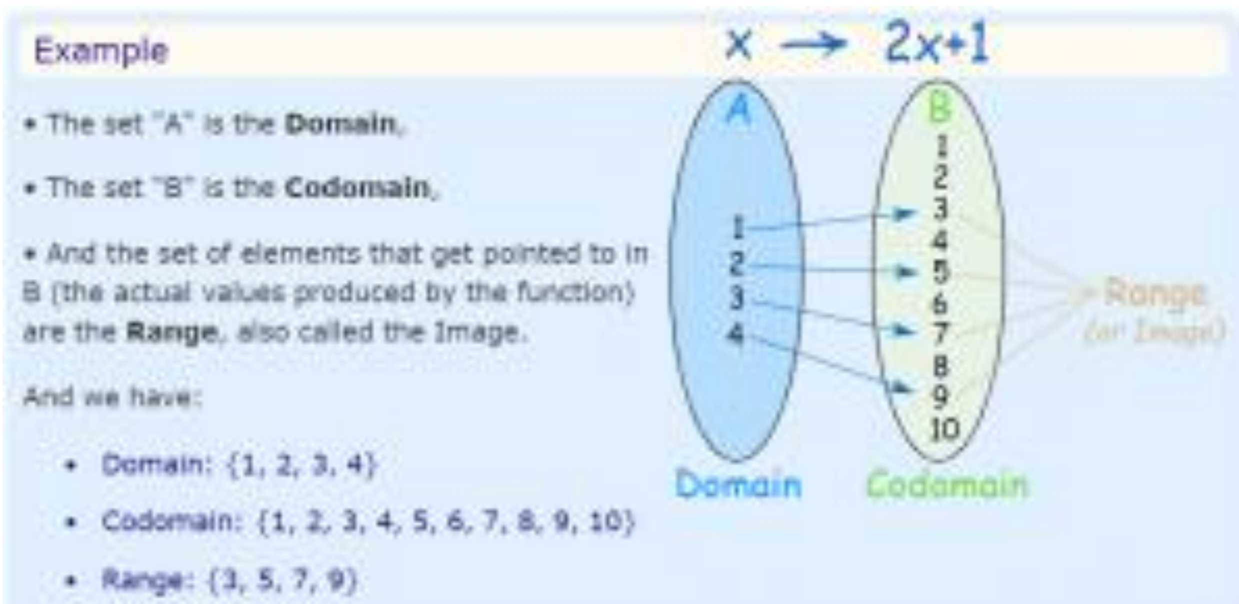


domain and range

Here, Domain = {1,5,8}

Range = {a,b,c}

<u>This will help you more:</u>



co-domain

Note: All ranges are co-domain but all co-domain are not range.

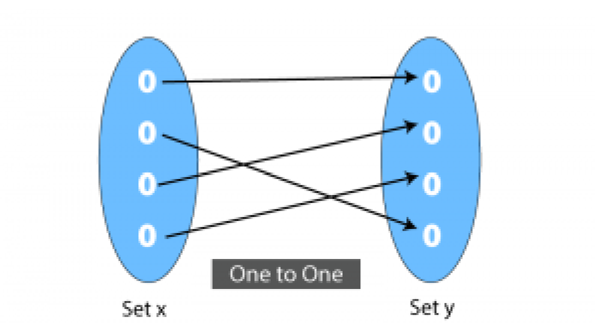
<u>Types of functions</u>

There are various types of functions in mathematics some of which are explained below in detail. The different functions types covered here are:

- One – one function (Injective function)
- Many – one function
- Onto – function (Surjective Function)
- Into – function
- Polynomial function
- Linear Function
- Quadratic Function
- Algebraic Functions
- Cubic Function
- Even and Odd Function
- Composite Function
- Constant Function
- Identity Function

One – one function (Injective function)

If each element in the domain of a function has a distinct image in the co-domain, the function is said to be one to one function.

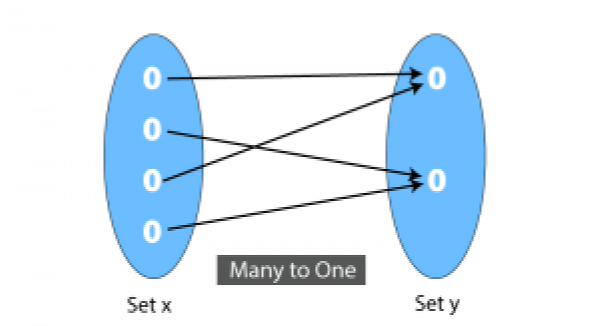


one-one function

For examples f: R R given by f(x) = 3x + 5 is one – one.

Many – one function

On the other hand, if there are at least two elements in the domain whose images are the same, the function is known as <u>many to one function.</u>



many-one function

Onto – function (Surjective Function)

A function is called an onto function if each element in the co-domain has at least one pre-image in the domain.

Into – function

If there exists at least one element in the co-domain which is not an image of any element in the domain then the function will be Into function.

Linear Function

All functions in the form of ax + b where a, b∈R & a ≠ 0 are called as linear functions. The graph will be a straight line. In other words, a linear polynomial function is a first-degree polynomial where the input needs to be multiplied by m and added to c. It can be expressed by f(x) = mx + c.

For example, f(x) = 2x + 1 at x = 1

f(1) = 2.1 + 1 = 3

f(1) = 3

Cubic Function

A cubic polynomial function is a polynomial of degree three and can be expressed as;

F(x) = ax3 + bx2 + cx + d and a is not equal to zero.

Quadratic Function

All functions in the form of y = ax2 + bx + c where a, b, c∈R, a ≠ 0 will be known as Quadratic function.

Algebraic Functions

A function that consists of a finite number of terms involving powers and roots of independent variable x and fundamental operations such as addition, subtraction, multiplication, and division is known as an algebraic equation.

Even and Odd Function

If f(x) = f(-x) then the function will be even function & f(x) = -f(-x) then the function will be odd function

Example 1:

f(x) = x2sinx f(x) = -f(-x)

f(-x) = -x2sinx

it is odd function.

Example 2:

f(x)=x2 ⇒f(x)=f(−x) f(−x)=x2 it is even function.

Periodic Function

A function is said to be a periodic function if there exists a positive real numbers T such that f(u – t) = f(x) for all x ε Domain.

For example f(x) = sinx

f(x + 2π) = sin (x + 2π) = sinx fundamental

then period of sinx is 2π

Composite Function

Let A, B, C' be three non-empty sets

Let f: A B & g : G C be two functions then gof : A C. This function is called composition of f and g

given g of (x) = g(f(x))

For example f(x) = x2 & g(x) = 2x

f(g(x)) = f(2x) = (2x)2 = 4x2

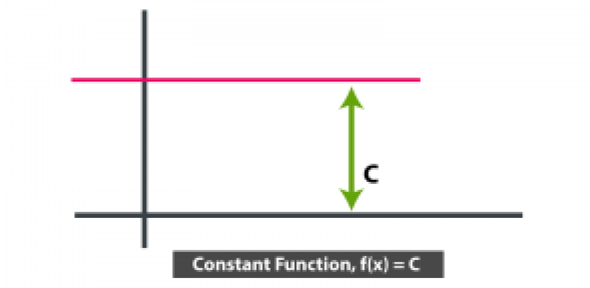
g(f(x)) = g(x2) = 2x2

Constant Function

The function **f : P → P** defined by **b = f (x) = D**, **a** ϵ**P**, where **D** is a constant ϵ **P**, is a constant function.

- Domain of **f = P**
- Range of **f = {D}**
- Graph type: A straight line which is parallel to the x-axis.

In simple words, the polynomial of 0th degree where f(x) = f(0) = a0=c. Regardless of the input, the output always results in constant value. The graph for this is a horizontal line.



Constant-Function

Identity Function

P= set of real numbers

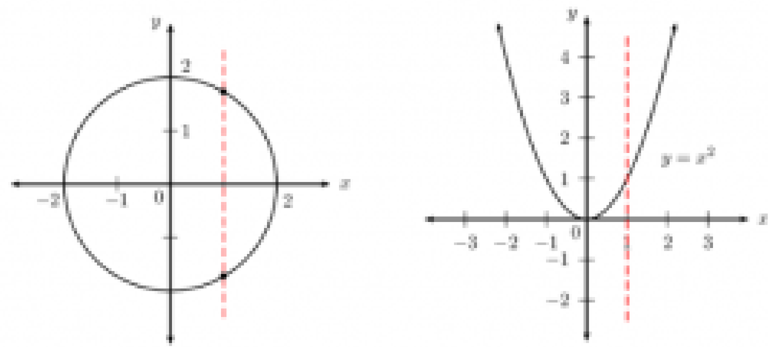
The function **f : P → P** defined by **b = f (a) = a** for each a ϵ **P** is called the identity function.

- Domain of **f** = **P**
- Range of **f** = **P**
- Graph type: A straight line passing through the origin.

Vertical line test

Given the graph of a relation, there is a simple test for whether or not the relation is a function. This test is called the vertical line test. If it is possible to draw any vertical line (a line of constant \(x\)) which crosses the graph of the relation more than once, then the relation is not a function. If more than one intersection point exists, then the intersections correspond to multiple values of \(y\) for a single value of \(x\) (one-to-many).

- If any vertical line cuts the graph only once, then the relation is a function (one-to-one or many-to-one).
- The red vertical line cuts the circle twice and therefore the circle is not a function.
- The red vertical line only cuts the parabola once and therefore the parabola is a function.



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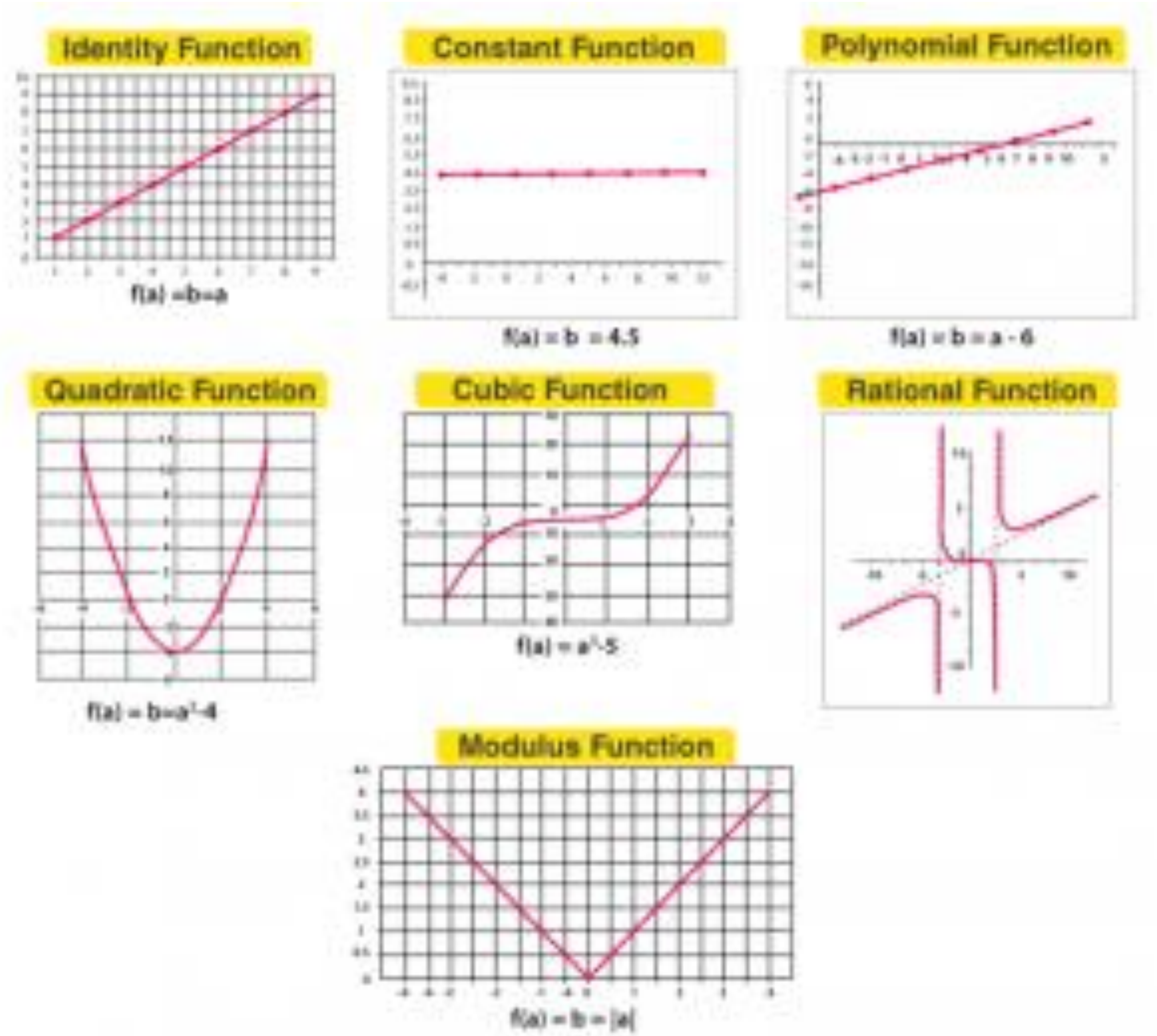


graphs

Unit 3. Sequence and Series

Arithmetic Progression (AP)

- An arithmetic progression is a sequence of numbers in which each term after the first is obtained by adding a constant ‘d’ to the preceding term. The constant d is called the common difference.

- An arithmetic progression is given by a, (a + d), (a + 2d), (a + 3d), …

where a = the first term, d = the common difference

- If a, b, c are in AP then b = (a + c)/2
- nth term of an arithmetic progression

tn = a + (n – 1)d

where tn = nth term, a= the first term, d= common difference

- Number of terms of an arithmetic progression

n=(*l*-a)/d+1

where n = number of terms, a= the first term , l = last term, d= common difference

-

FORMULAS

Number of terms of an arithmetic progression

$$n = \frac{(l - a)}{d} + 1$$

where n = number of terms, a= the first term , l = last term, d= common difference

no of term

Sum of first n terms in an arithmetic progression

$$S_n = \frac{n}{2}[\,2a + (n-1)d\,] = \frac{n}{2}(a + l)$$

where a = the first term,

d= common difference,

$l = t_n = n^{th}$ term = a + (n-1)d

the sum of first n terms

ADDITIONAL NOTES ON AP

To solve most of the problems related to AP, the terms can be conveniently taken as

3 terms: (a – d), a, (a +d)

4 terms: (a – 3d), (a – d), (a + d), (a +3d)

5 terms: (a – 2d), (a – d), a, (a + d), (a +2d)

Harmonic Progression(HP)

Harmonic Progression(HP)

Non-zero numbers $a_1, a_2, a_3, \cdots a_n$ are in Harmonic Progression(HP) if $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \cdots \frac{1}{a_n}$ are in AP. Harmonic Progression is also known as harmonic sequence.

Examples

$\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \cdots$ is a harmonic progression (HP)

Three non-zero numbers a, b, c will be in HP, if $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP

If a, (a+d), (a+2d), . . . are in AP, n^{th}term of the AP = a + (n - 1)d

Hence, if $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \cdots$ are in HP, n^{th}term of the HP = $\frac{1}{a+(n-1)d}$

harmonic progression

If a, b, c are in HP, b is the Harmonic Mean(HM) between a and c

In this case, $b = \frac{2ac}{a+c}$

The Harmonic Mean(HM) between two numbers a and b = $\frac{2ab}{a+b}$

If a, a_1, a_2 ... a_n, b are in HP we can say that a_1, a_2 ... a_n are the n Harmonic Means between a and b.

If a, b, c are in HP, $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

harmonic progression

Geometric Progression (GP)

Geometric Progression (GP) is a sequence of non-zero numbers in which the ratio of any term and its preceding term is always constant.

A geometric progression(GP) is given by a, ar, ar^2, ar^3, ...
where a = the first term , r = the common ratio

Examples

1, 3, 9, 27, ... is a geometric progression(GP) with a = 1 and r = 3

2, 4, 8, 16, ... is a geometric progression(GP) with a = 2 and r = 2

If a, b, c are in GP, $b^2 = ac$

geometric progression

n^{th} term of a geometric progression(GP)

$$t_n = ar^{n-1}$$

where t_n = n^{th} term, a= the first term , r = common ratio, n = number of terms

Sum of first n terms in a geometric progression(GP)

$$S_n = \begin{cases} \dfrac{a(r^n - 1)}{r - 1} & (\text{if } r > 1) \\ \dfrac{a(1 - r^n)}{1 - r} & (\text{if } r < 1) \end{cases}$$

where a= the first term,
r = common ratio,
n = number of terms

sum of GP

Sum of an infinite geometric progression(GP)

$$S_\infty = \frac{a}{1 - r} \quad (\text{if } \ -1 < r < 1)$$

where a= the first term , r = common ratio

sum of infinite

Additional Notes on GP

To solve most of the problems related to GP, the terms of the GP can be conveniently taken as

3 terms: $\frac{a}{r}$, a, ar

5 terms: $\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar^2

If a, b, c are in GP, $\dfrac{a - b}{b - c} = \dfrac{a}{b}$

In a GP, product of terms equidistant from beginning and end will be constant.

geometric progression

Power Series : Important formulas

$$1 + 1 + 1 + \cdots \text{ n terms} = \sum 1 = n$$

$$1 + 2 + 3 + \cdots + n = \sum n = \frac{n(n+1)}{2}$$

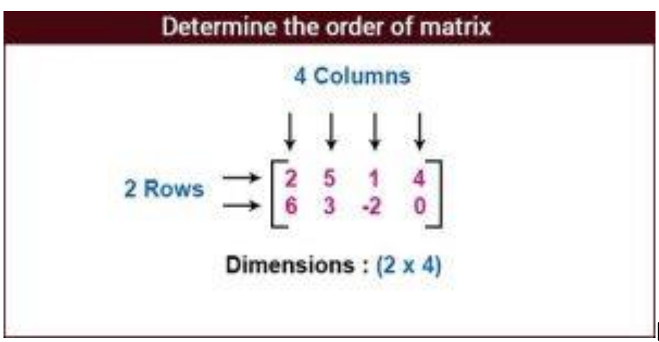
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \sum n^3 = \frac{n^2(n+1)^2}{4} = \left[\frac{n(n+1)}{2}\right]^2$$

power series

Unit 4: Matrix and Determinants

A **matrix** is simply a set of numbers arranged in a rectangular table.



Determine-the-order-of-matrix

Types of Matrices

There are several types of matrices, but the most commonly used are:

- Rows Matrix
- Columns Matrix
- Rectangular Matrix

- Square Matrix
- Diagonal Matrix
- Scalar Matrix
- Identity Matrix
- Triangular Matrix
- Null or Zero Matrix
- Transpose of a Matrix

Row Matrix:
A matrix is said to be a row matrix if it has only one row.

$$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

row matrix

Column Matrix:
A matrix is said to be a column matrix if it has only one column.

$$B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

row matrix

Rectangular Matrix:
A matrix is said to be rectangular if the number of rows is not equal to the number of columns.

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 2 \end{bmatrix}$$

rectangular matrix

Square Matrix:
A matrix is said to be square if the number of rows is equal to the number of columns.

$$B = \begin{bmatrix} 1 & 3 & 4 \\ 5 & 2 & 4 \\ 1 & 9 & 6 \end{bmatrix}$$

square matrix

Diagonal Matrix:
A square matrix is said to be diagonal if at least one element of principal diagonal is non-zero and all the other elements are zero.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

diagonal matrix

Scalar Matrix:
A diagonal matrix is said to be scalar if all of its diagonal elements are the same.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

scalar matrix

Identity or Unit Matrix:
A diagonal matrix is said to be identity if all of its diagonal elements are equal to one, denoted by I.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

identity matrix

Triangular Matrix:
A square matrix is said to be triangular if all of its elements above the principal diagonal are zero **(lower triangular matrix)** or all of its elements below the principal diagonal are zero **(upper triangular matrix)**.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix} \backslash[(Lower Triangular Matrix)\backslash[B = \begin{bmatrix} 5 & 8 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 7 \end{bmatrix}$$

traingular

Null or Zero Matrix:

A matrix is said to be a null or zero matrix if all of its elements are equal to zero. It is denoted by O.

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \; O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

zero matrix

Transpose of a Matrix:

Suppose A is a given matrix, then the matrix obtained by interchanging its rows into columns is called the transpose of A. It is denoted by At.

e.g.
Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 8 & 4 & 6 \end{bmatrix}$$

then

$$A^t = \begin{bmatrix} 1 & 8 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$$

transpose matrix

<u>Determinants of a matrix</u>

The determinant of a matrix is a **special number** that can be calculated from a square matrix.

For a 2×2 Matrix

<u>For a 2×2 matrix (2 rows and 2 columns):</u>

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The determinant is:

$$|A| = ad - bc$$

"The determinant of A equals a times d minus b times c"

determinant

$$B = \begin{bmatrix} 4 & 6 \\ 3 & 8 \end{bmatrix}$$

$$\begin{aligned} |B| &= 4\times8 - 6\times3 \\ &= 32-18 \\ &= 14 \end{aligned}$$

For a 3×3 Matrix

For a 3×3 matrix (3 rows and 3 columns):

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

The determinant is:

$$|A| = a(ei - fh) - b(di - fg) + c(dh - eg)$$

"The determinant of A equals ... etc"

It may look complicated, but **there is a pattern**:

To work out the determinant of a **3×3** matrix:

- Multiply **a** by the **determinant of the 2×2 matrix** that is **not in a**'s row or column.
- Likewise for **b**, and for **c**
- Sum them up, but remember the minus in front of the **b**

Adjoint of a Matrix

Let A=[aij] be a square matrix of order n. The adjoint of matrix A is the transpose of the cofactor matrix of A. It is denoted by adj A. An adjoint matrix is also called an adjoint matrix.

Example:

Example:

Find the adjoint of the matrix.

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

eg of the cofactor matrix

To find the adjoint of a matrix, first find the cofactor matrix of the given matrix. Then find the transpose of the cofactor matrix.

Cofactor of $3 = A_{11} = \begin{vmatrix} -2 & 0 \\ 2 & -1 \end{vmatrix} = 2$

Cofactor of $1 = A_{12} = -\begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} = 2$

Cofactor of $-1 = A_{13} = \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix} = 6$

Cofactor of $2 = A_{21} = -\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1$

Cofactor of $-2 = A_{22} = \begin{vmatrix} 3 & -1 \\ 1 & -1 \end{vmatrix} = -2$

Cofactor of $0 = A_{23} = -\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = -5$

Cofactor of $1 = A_{31} = \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = -2$

Cofactor of $2 = A_{32} = -\begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} = -2$

Cofactor of $-1 = A_{33} = \begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix} = -8$

The cofactor matrix of A is $[A_{ij}] = \begin{bmatrix} 2 & 2 & 6 \\ -1 & -2 & -5 \\ -2 & -2 & -8 \end{bmatrix}$

Now find the transpose of A_{ij} .

$$adj\, A = (A_{ij})^T$$

$$= \begin{bmatrix} 2 & -1 & -2 \\ 2 & -2 & -2 \\ 6 & -5 & -8 \end{bmatrix}$$

<u>Inverse of a matrix</u>

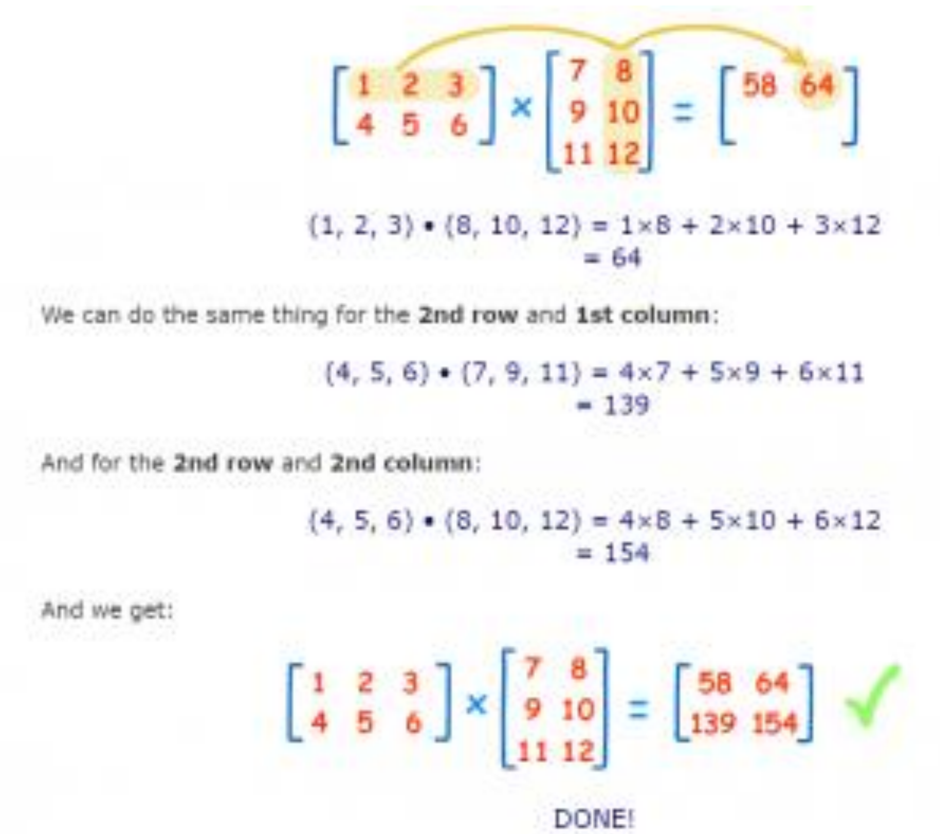
The inverse of A is A-1 only when:

A × A-1 = A-1 × A = **I**

Sometimes there is no inverse at all.

<u>Formula:</u>

<u>Multiplication of a matrix</u>



matrix multiplication

Unit 5: Analytical Geometry

Analytic Geometry is a branch of algebra that is used to model geometric objects – points, (straight) lines, and circles being the most basic of these. Analytic geometry is a great invention of Descartes and Fermat.

In plane analytic geometry, points are defined as ordered pairs of numbers, say, (x, y), while the straight lines are in turn defined as the sets of points that satisfy linear equations, see the excellent expositions by D. Pedoe or D. Brannan et al. From the view of analytic geometry, geometric axioms are derivable theorems. For example, for any two distinct points (x1, y1) and (x2, y2), there is a single line ax + by + c = 0 that passes through these points. Its coefficients a, b, c can be found (up to a constant factor) from the linear system of two equations

ax1 + by1 + c = 0
ax2 + by2 + c = 0,

1. Conic Section
2. Vector in space

Unit 6: Permutation and Combination

What's the Difference?

- When the order doesn't matter, it is a **Combination**.
- When the order **does**matter it is a **Permutation**.

In other words:

A Permutation is an **ordered** Combination.

Permutations

There are basically two types of permutation:

- **Repetition is Allowed**: It could be "333".
- **No Repetition**: for example, the first three people in a running race. You can't be first *and*

1. Permutations with Repetition

When a thing has ***n*** different types … we have ***n*** choices each time!

For example: choosing ***3*** of those things, the permutations are:

n × n × n*(n multiplied 3 times)*

Example: in the lock above, there are 10 numbers to choose from (0,1,2,3,4,5,6,7,8,9) and we choose 3 of them:

10 × 10 × 10 (3 times) = 103 = 1,000 permutations

So, the formula is simply:

nr

---

where ***n*** is the number of things to choose from,
and we choose ***r*** of them,
repetition is allowed,
and order matters.

---

2. Permutations without Repetition

In this case, we have to **reduce** the number of available choices each time.

Example: what order could 16 pool balls be in?

After choosing, say, number "14" we can't choose it again.

So, our first choice has 16 possibilites, and our next choice has 15 possibilities, then 14, 13, 12, 11, ... etc. And the total permutations are:

16 × 15 × 14 × 13 × ... = 20,922,789,888,000

But maybe we don't want to choose them all, **just 3** of them, and that is then:

16 × 15 × 14 = 3,360

In other words, there are 3,360 different ways that 3 pool balls could be arranged out of 16 balls.

But how do we write that mathematically? Answer: we use the " factorial function "

The **factorial function** (symbol: **!**) just means to multiply a series of descending natural numbers. Examples:

- 7! = 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5,040
- 1! = 1

What's the Difference?

- When the order doesn't matter, it is a **Combination**.
- When the order **does**matter it is a **Permutation**.

In other words:

A Permutation is an **ordered** Combination.

Permutations

There are basically two types of permutation:

- **Repetition is Allowed**: It could be "333".
- **No Repetition**: for example, the first three people in a running race. You can't be first *and*

1. Permutations with Repetition

When a thing has ***n*** different types … we have ***n*** choices each time!

For example: choosing ***3*** of those things, the permutations are:

n × n × n*(n multiplied 3 times)*

Example: in the lock above, there are 10 numbers to choose from (0,1,2,3,4,5,6,7,8,9) and we choose 3 of them:

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So, when we want to select **all** of the pool balls the permutations are:
16! = 20,922,789,888,000

But when we want to select just 3 we don't want to multiply after 14. How do we do that? Simple,we divide by **13!**

***16 × 15 × 14 × 13 × 12 …*13 × 12 … = 16 × 15 × 14**

Hint:
The **13 × 12 × … etc** gets "cancelled out", leaving only **16 × 15 × 14**.

The formula is written:

n!(n − r)!
where ***n*** is the number of things to choose from, and we choose ***r*** of them, no repetitions, order matters.

This will help you :

Example: Our "order of 3 out of 16 pool balls example" is:

$$\frac{16!}{(16-3)!} = \frac{16!}{13!} = \frac{20{,}922{,}789{,}888{,}000}{6{,}227{,}020{,}800} = 3{,}360$$

(which is just the same as: **16 × 15 × 14 = 3,360**)

Notation

Instead of writing the whole formula, people use different notations such as these:

$$P(n,r) = {}^nP_r = {}_nP_r = \frac{n!}{(n-r)!}$$

Example: *P*(10,2) = 90

Combinations

There are also two types of combinations (remember the order does **not** matter now):

- **Repetition is Allowed**: such as coins in your pocket (5,5,5,10,10)
- **No Repetition**: such as lottery numbers (2,14,15,27,30,33)

Notation

As well as the "big parentheses", people also use these notations:

$$C(n,r) = {}^{n}C_{r} = {}_{n}C_{r} = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

(It is often called "n choose r" (such as "16 choose 3")

And is also known as the Binomial Coefficient.)

1. Combinations with Repetition

$$\binom{r+n-1}{r} = \frac{(r+n-1)!}{r!(n-1)!}$$

where n is the number of things to choose from,
and we choose r of them
repetition allowed,
order doesn't matter.

Example:

Let us say there are five flavors of ice-cream: **banana, chocolate, lemon, strawberry and vanilla**.

We can have three scoops. How many variations will there be?

So, here **n=5** and **r=3, using formula**

$$\frac{(3+5-1)!}{3!(5-1)!} = \frac{7!}{3!\times 4!} = \frac{5040}{6\times 24} = 35$$

There are 35 ways of having 3 scoops from five flavors of ice-cream.

2. Combinations without Repetition

Example: Pool Balls (without order)

So, our pool ball example (now without order) is:

16! **3!(16−3)!** = *16!* **3! × 13!**

= *20,922,789,888,000* **6 × 6,227,020,800**

= 560

Or we could do it this way:

16×15×14 **3×2×1** = *3360* **6** = 560

Notation

Instead of writing the whole formula, people use different notations such as these:

$$P(n,r) = {}^{n}P_{r} = {}_{n}P_{r} = \frac{n!}{(n-r)!}$$

Example: *P*(10,2) = 90

Combinations

There are also two types of combinations (remember the order does **not** matter now):

- **Repetition is Allowed**: such as coins in your pocket (5,5,5,10,10)
- **No Repetition**: such as lottery numbers (2,14,15,27,30,33)

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As well as the "big parentheses", people also use these notations:

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$$\frac{16!}{3!(16-3)!} = \frac{16!}{3! \times 13!}$$

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$$= 560$$

Or we could do it this way:

$$\frac{16 \times 15 \times 14}{3 \times 2 \times 1} = \frac{3360}{6} = 560$$